Effects of Thermal Discharges on the Stratification Cycle of Lakes

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The effects of power-plant thermal discharges on the seasonal stratification cycle of temperate lakes are considered. A theoretical model, which includes the interaction between wind-induced turbulence in the lake and the buoyancy gradients due to surface heating as well as the thermal discharge, is developed and solved numerically for some typical cases. It is shown that the thermal discharges increase the temperature of the epilimnion, the temperature during spring homothermy, and the stratification.

Introduction

IN recent years increasing concern has been expressed over the possible adverse ecological consequences of the discharge of waste heat from electric power generating plants into various bodies of water.[1,2] A necessary prerequisite to the assessment of the ecological consequences of such "thermal pollution" is the assessment of the effects of the discharges on the physical parameters of the body of water. The present paper describes the results of a study of the effects of power-plant thermal discharges on the seasonal stratification cycle of deep, temperate lakes. Such a quantitative understanding of the relationship between the seasonal variations of the thermal structure of a lake and the environmental conditions above it is essential before the perturbing effects of the added heat load on the lake can be assessed; the present paper also describes briefly studies of the basic stratification cycle. A more complete discussion of a theory for the stratification cycle of temperate lakes has been given elsewhere.[3]

When heated effluents are discharged into a large body of water such as a stratified lake, the turbulent dispersion of heat takes place over two, more or less distinct scales (both in a spatial and temporal sense).[2,4,5] The first scale is concerned with the dispersion of the effluents in some vicinity of the outfall where a distinct thermal plume can be identified. It is in this thermal-plume region that the initial distribution of heat into the lake takes place and, for a suracing discharge, a direct loss of some of the heat to the atmosphere also occurs. The second scale, which is of much larger spatial and temporal extent than the first, is concerned with the effects of the thermal discharges on the entire lake including those on the seasonal stratification cycle. These two distinct scales have been termed micro and macro scales by some authors[2] and near-field and far-field scales by other.[4,5]

The major advantage of dividing the problem into separate scales is that different approximations can be used to study different scales. Thus when treating the thermal-plume problem the ambient conditions, including the atmosphere conditions above the lake, can be considered to be steady. On the other hand, in the far-field problem conditions are assumed to be homogeneous in planes parallel to the surface of the lake, and the effects of vertical diffusion and the seasonal changes in conditions above the lake are studied. The physical justification for the assumption of horizontal homogeneity is that horizontal mixing processes are in general much faster than vertical ones; that is, the characteristic time scales for horizontal mixing are much smaller than those for the vertical mixing as well as those for the seasonal changes in conditions above the lake.

The present paper describes a theory for the effects of power plant thermal discharges on the seasonal stratification cycle of temperate lakes. In the next section, a theoretical description for the stratification cycle of temperate lakes, which has been described by the authors elsewhere,[3] is reviewed briefly. The theory is then modified to include the effects of thermal discharges at or below the level of the thermocline, and specific numerical examples are presented. It should be emphasized that, when considering the effects of thermal discharges on the stratification cycle, it is important to account properly for the interaction between the turbulence and thermal structures. Heated effluents not only influence the thermal structure directly, but they also influence it indirectly through their influence on the structure of the turbulence.[5] Thus, if the effects of the thermal discharges on the mechanisms of mixing are altogether neglected (a step which cannot be justified), then the problem of predicting the effects of thermal discharges on the lake becomes relatively simple. There are no existing theories which account for the changes, due to the thermal discharges, in the eddy diffusivities in a stratified lake.

Stratification Cycle

Qualitative features of the stratification cycle of temperate lakes have been described by Hutchison[6] and Ruttner.[7] During early spring, most temperate lakes exhibit a nearly homothermal state with a temperature of about 4°C (which is the temperature of maximum density for water) extending from top to bottom. As the lake begins to warm, initially the temperature decreases smoothly from top to bottom but, at later times, a point of inflection, or thermocline, develops in the temperature profile. The thermocline separates the well-mixed upper layers of the lake (the epilimnion) from the deeper regions of the lake (the hypolimnion). By the time the lake attains its maximum heat content, a well-mixed upper layer, with a sharply defined lower boundary, already exists. As the lake begins to cool, the thermocline descends rapidly into the deeper layers of the lake and, when it reaches the bottom, the lake once again attains homothermy. The lake then cools uniformly to its minimum temperature and the cycle is repeated the next year.

In an earlier paper,[8] we have shown that the thermocline is formed by the nonlinear interaction between wind-induced turbulence and buoyancy gradients due to surface heating. In Ref. 8, the influence on the eddy diffusivities of the interaction between the turbulence structure and the buoyancy gradients was included by using the techniques that have been successful in the study of atmospheric turbulence.[9] That is, the eddy diffusivities were assumed to be given by the product of the eddy diffusivities under conditions of neutral stability and an appropriate function of a stratification parameter such as the gradient

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Richardson number.[10] The nonlinear equations of the problem were then solved, using an electronic computer, for the simple case of a half-space, which is initially at a uniform temperature, and then subjected to a sudden increase in surface temperature. For this case, the temperature distributions were initially similar to the familiar error-function distribution for the linear case, but nonlinear effects soon came into play, and a thermocline was formed some distance below the surface. The thermocline also propagated steadily away from the surface.

The theoretical model developed by Sundaram and Rehm[8] can also be used to study the entire stratification of temperate lakes. However, when considering the stratification cycle, in addition to including the effects of the interaction between the wind-induced turbulence and buoyancy gradients due to surface heating, the occurrence of free convection during the cooling part of the cycle and the variation of the volumetric coefficient of expansion of water with temperature should also be included in the analysis.

The basic relations used in Ref. 8 are

$$\partial T/\partial t = (\partial/\partial z)(K_H \partial T/\partial z) \quad (1)$$

$$K_H = K_{H_0}(1+\sigma R_i)^{-1} \quad (2)$$

$$R_i = -\alpha_v g z^2 (\partial T/\partial z)/w^{*2} \quad (3)$$

In the previous equations, T is the temperature, t is the time, z is the distance measured downward from the surface, K_H is the eddy diffusivity for the transport of heat under arbitrary stratification, K_{H_0} is the eddy diffusivity under neutral conditions, R_i is a special form of the Richardson number, α_v is the volumetric coefficient of expansion of water and $w^* = \tau_s/\rho$ is the friction velocity where τ_s is the surface shear stress induced by the wind and ρ is the density of water.

The form of the eddy diffusivity in Eq. (2) was first proposed by Rossby and Montgomery,[11] whereas the form of the Richardson number given in Eq. (3) implies that the mechanical generation of turbulence in the upper layers of a lake can be expressed directly in terms of the wind stress without explicit consideration of the current structure.[3,8] However, after the onset of stratification, an implicit accounting is necessary for the change in the nature of the current structure and the attendant change in the character of the turbulence in the deeper layers. Also, when the stratification in the upper layers of the lake become sufficiently unstable (as in the cooling part of the cycle), Eq. (2) will become invalid in these layers due to onset of free convection. The appropriate procedures that are required to treat the above special cases have been described fully in Ref. 3.

When solving Eqs. (1–3) for the entire stratification cycle, the necessary boundary conditions at the surface have to be specified in terms of the known variations in the appropriate environmental conditions above the lake. In other words, the thermal structure of the lake has to be viewed in terms of certain "external parameters" which specify the exchange of thermal and mechanical energies between the lake and the environment. The exchange of thermal energy can be specified in terms of an equilibrium temperature, which is defined to be the fictitious lake-surface temperature at which there is no heat exchange between the lake and the environment.[12,3,5] That is,

$$q_s = -(\rho C_p K_H \partial T/\partial z)_{z=0} = K(T_E - T_S) \quad (4)$$

where q_s is the heat flux at the surface, C_p is the specific heat, K is a heat-exchange coefficient, T_s is the surface temperature and T_E is the equilibrium temperature. The equilibrium temperature is, in general, a function of the incoming (sky and solar) radiation, wind speed, air temperature and humidity.[12,5] Methods of determining T_E and K from the environmental conditions have been described by Edinger and Geyer[12] and Sundaram et al.[5]

The annual variation of the equilibrium temperature over most temperate lakes can be represented[5] by the simple sinusoidal relation,

$$T_E = \bar{T}_E + \delta T_E \sin(\omega t + \phi) \quad (5)$$

where $\bar{T}_E$ is the average value of the equilibrium temperature over one annual cycle, δT_E is one half of the annual variation and $\omega = (2\pi/365)$ days^{-1}. The value of ϕ will depend upon the

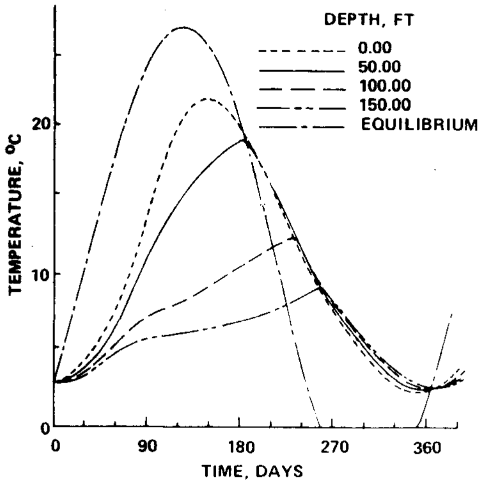

Fig. 1 The stratification cycle.

conditions from which the analysis is begun. The annual variations in the parameter K can also be represented in the same manner as in Eq. (5). However, in most cases the annual variation of K is small, and K can be taken, to a sufficient accuracy, as a constant.

The exchange of mechanical energy between the lake and the atmosphere is determined by specifying the wind conditions above the lake. The friction velocity w^* and the eddy diffusivity under neutral conditions K_{H_0} can be related to the wind speed by using semiempirical relations such as those given by Munk and Anderson.[13] In the present paper, the friction velocity has been assumed to be a cyclic function of time of the form,

$$w^* = B_1 + B_2 \sin(\omega t + \psi) \quad (6)$$

where B_1, B_2, and ψ are constants to be determined from the known wind conditions above the lake. A similar relation has been assumed for the variation of K_{H_0} also.

The basic equations and boundary conditions represented by Eqs. (1–6) have been solved numerically using an explicit finite difference scheme. The features of the solutions for various input conditions have been described elsewhere,[3] and they will not be discussed here since the primary purpose of the present paper is to discuss the effects of thermal discharges on the stratification cycle. However, in order to facilitate the interpretation of the effects of thermal discharges, a brief description of some of the key features of the basic solution (that is, the solution with no thermal discharges) are given here. Some typical computer-generated results for the basic stratification cycle are shown in Figs. 1 and 2. The input conditions for the calculation shown correspond, roughly, to those for Cayuga Lake,[5,14] New York. Figure 1 shows five temperatures, the equilibrium temperature (which is an input condition for the calculation), the surface temperature and the temperatures at depths of 50, 100, and 150 ft, as functions of time for approximately one year. Figure 2 displays the temperature as a function of depth at various times during the annual cycle. The

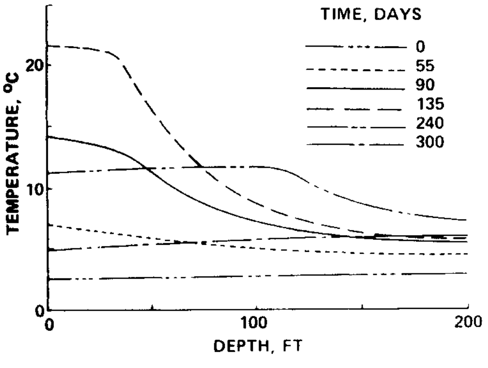

Fig. 2 Vertical temperature distributions.

lake was assumed to be homothermal initially at 2.9°C, corresponding to conditions in March. It can be seen from Fig. 1 that stratification occurs at around 60 days. The depth of the thermocline decreases to a minimum value of about 35 ft between 120 and 150 days, and then increases rapidly as the cooling part of the cycle begins. The thermocline reaches the bottom, and the stratification ends, at about 270 days.

Even from the brief description given above it can be seen that the present theory predicts all the observed characteristic features of the stratification cycle of temperature lakes. For example, the present theory predicts the formation of the thermocline some time after the period of maximum spring homothermy (with an intermediate period of smoothly varying temperature distributions), but before the lake attains its maximum heat content and begins to lose heat to the atmosphere. Also, the surface temperature of the lake attains a maximum before the heat content attains a maximum.

In the following sections the modifications of Eqs. (1–6) that are necessary to account for the effects of thermal discharges below the thermocline are described.

Effects of Thermal Discharges

An important problem that arises in connection with thermal pollution is the effect of thermal discharges into a stratified body of water at or below the thermocline level. Discharges below the thermocline have been used extensively in connection with sewage disposal into marine environments.[15–17] The primary advantage of this mode of discharge is that, because of sewage being heavier than the surface sea water, the sewage field can be held submerged below the thermocline thereby preventing contamination of the shore areas. In this section we will consider the effects of thermal discharges from power plants for the case when the intake water is withdrawn from the hypolimnion, and the heated effluents are injected back into the hypolimnion in such a manner as to trap them below the thermocline during part of the stratification period.

When thermal discharges are injected into the hypolimnion of a stratified body of water, they will rise as a buoyant plume because of their positive buoyancy with respect to the surrounding cooler medium (see Fig. 3). As the plume rises, it will entrain the cooler surrounding water into it. The entrainment will increase the volume of the plume and also reduce the temperature difference between the plume and the surroundings. In addition, the temperature difference between the plume and the ambient water also decreases due to the fact that the ambient temperature increases with increasing distance from the point of discharge. Even after the plume becomes neutrally buoyant, it will continue to rise because of its residual kinetic energy and will come to rest only when this energy is fully dissipated. After this point the plume will spread rapidly in the horizontal direction as a relatively thin sheet. The initial horizontal spreading of the thermal plume will create a well-mixed layer which is nearly homothermal so that, due to the stable stratification at the (horizontal) boundaries of this layer, vertical mixing will be inhibited.

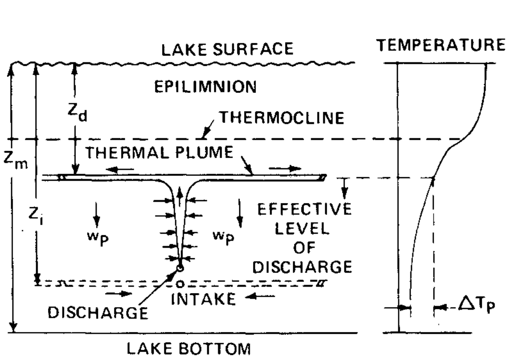

Fig. 3 Schematic representation of the thermal plume.

Recently Baines and Turner[18] have studied, both experimentally and theoretically, the effects of continuous convection from a small source of buoyancy enclosed in a bounded region. They point out that conservation of mass requires that, at any horizontal plane, the upward volume flux in the plume be balanced by an equal downward flux in the surrounding region. Also, because of the stable stratification in the surrounding region, the fluid that is entrained into the plume at any level (including the intake level) can be assumed to come entirely from that level. In the present paper, these concepts have been adapted to study the lake-wide effects of thermal discharges into the hypolimnion of a stratified lake. However, a detailed consideration of the behavior of the buoyant plume itself has not been included. Rather, the effluents have been assumed to be injected directly into the level at which the lake temperature is equal to the discharge temperature. It is planned to include the effects of dilution in future modifications of the present model using existing knowledge[18–20] on the behaviors of buoyant plumes in density stratified environments.

In the present study, following Baines and Turner,[18] it has been assumed that the effluents spread instantaneously into a thin sheet at their level of neutral buoyancy, and the details of the actual horizontal spread have been ignored. This assumption is justified in view of the relatively long time scales involved in the consideration of the lake-wide effects of the discharges. In the present model, the effect of the intake will be equivalent to that of a uniformly distributed sink at the level of the intake, with the total strength of the sink corresponding to the volumetric flow rate through the power-plant condensors. There will be a corresponding uniformly distributed source of fluid at the effective level of injection of the discharge.

When considering the effects of thermal discharges from a power plant with a hypolimnetic intake, in addition to the effect associated with the discharge of heat there is also an effect associated with the transfer of large quantities of water from one level to another. This latter effect arises due to a change in the potential energy of the stratification and, as pointed out in Ref. 5, for a surface discharge it can be viewed in terms of an equivalent change in wind conditions above the lake. In the present model the effect of "pumping" water from a lower level to a higher one has been accounted for properly.

Although there are different ways in which the effects of heat and pumping work could be introduced, when the effluent is discharged below the surface, these effects will be included in Eq. (1) through the addition of a single term $w_p \partial T/\partial z$. Here w_p is the uniform, downward vertical velocity which is induced (between the level of the intake and the effective level of the discharge) by the power plant pumping; w_p can be assumed to be the volumetric flow rate Q_p through the power plant divided by the area A of the lake. With the addition of this term Eq. (1) becomes

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K_H \frac{\partial T}{\partial z}\right) - w_p \frac{\partial T}{\partial z} \qquad z_i \geqq z \geqq z_d \tag{7}$$

If the level of the intake for water is fixed at z_i and the level of discharge of the heated effluent is at z_d (see Fig. 3), then Eq. (7) is appropriate for $z_i \geqq z \geqq z_d$. Below the intake level $z > z_i$ and above the discharge level $z < z_d$, the direct effects of the power-plant pumping are absent, and the governing equation is simply Eq. (1).

The meaning of the extra term in Eq. (7) can be understood by examining integrals of Eq. (7). Integrating these equations from the surface of the lake, $z = 0$, to the bottom of the lake, $z = z_m$ we obtain

$$\frac{d}{dt}\int_0^{z_m} T dz = \left(K_H \frac{\partial T}{\partial z}\right)_{z=z_m} - \left(K_H \frac{\partial T}{\partial z}\right)_{z=0} - w_p[T(z_i) - T(z_d)] \tag{8}$$

Multiplying by $\rho C_p A$ and noting the surface boundary condition given by Eq. (4) and the boundary condition that the heat flux at the bottom of the lake is zero, Eq. (8) can be written in the form

$$\frac{d}{dt}\int_0^{z_m} \rho C_p T A\, dz = AK(T_E - T_S) + Q_p \rho C_p [T(z_d) - T(z_i)] \equiv Aq_s + Aq_p \qquad (9)$$

Equation (9) is simply and directly interpretable. It states that the time rate of increase of thermal energy, integrated over the whole lake, is equal to the heat added through the surface Aq_s plus the heat added by the power plant Aq_p. Note that the heat added by the power plant is simply the water volumetric plant flow rate Q_p times the energy per unit volume required to heat water from the intake temperature $T(z_i)$ to the discharge temperature $T(z_d)$.

Taking the first moment of Eq. (7) and integrating again from $z = 0$ to $z = z_m$, we find, after some manipulation,

$$\frac{d}{dt}\int_0^{z_m} zT dz = \int_0^{z_m}\left(-K_H \frac{\partial T}{\partial z}\right)dz + w_p z_d [T(z_d) - T(z_i)] + w_p \int_{z_d}^{z_i} [T(z) - T(z_i)]\, dz \qquad (10)$$

Equation (10) can be interpreted in terms of the changes in the potential energy of stratification.

The potential energy of stratification of a lake can be written in the form

$$P = \rho \alpha_v g A \int_0^{z_m} (z - z_g)(T - \bar{T})\, dz \qquad (11)$$

where z_g is the depth of the center of gravity of the lake and $\bar{T}$ is the temperature the lake would attain at any given time if the wind energy were able to mix it completely. Note that the potential energy of stratification is always negative since wind energy is required to upset the stratification. Thus the stronger the stratification, the more negative will be the potential energy. The time rate of change of potential energy can be written as

$$\frac{dP}{dt} = \rho \alpha_v g A \frac{d}{dt}\int_0^{z_m} zT dz - \rho \alpha_v g A z_g \frac{d}{dt}\int_0^{z_m} T dz \qquad (12)$$

Equations (12, 9, and 10) can be combined to yield

$$\frac{dP}{dt} = -\rho \alpha_v g A z_g \frac{q_s}{\rho C_p} + \rho \alpha_v g A \int_0^{z_m}\left(-K_H \frac{\partial T}{\partial z}\right)dz + \rho \alpha_v g A(z_d - z_g)\frac{q_p}{\rho C_p} + \rho \alpha_v g A w_p \int_{z_d}^{z_i} [T(z) - T(z_i)]\, dz \qquad (13)$$

The first term in the right-hand side of Eq. (13) represents the change in the potential energy of stratification due to the surface heat flux, whereas the second term represents the change due to the heat flux in the deeper layers. Since the heat flux at the deeper layers arises out of the wind-induced mixing of the surface heat income, the second term can be identified with the effects of wind mixing. The third term in Eq. (13) represents the change in potential energy of stratification due to the thermal discharges. Note that it depends only on the amount of heat added and the effective level of the discharge. The fourth term in Eq. (13) represents the effect of pumping and it depends on the pumping velocity w_p.

When the temperature at which the effluent is discharged is higher than the temperature at any depth within the lake, then the effluent will surface and spread horizontally. The model discussed above is then no longer adequate, since in order for the term $w_p \partial T/\partial z$ in Eq. (7) to represent both the heat input and the pumping work the effluent and ambient temperatures at the discharge level must be equal. Under these conditions, then, the part of the power-plant waste heat in addition to that represented by the term $w_p \partial T/\partial z$ must be explicitly introduced into Eq. (7). If ΔT_p represents the temperature change produced in the intake water by the power plant, the additional heat per unit area per unit time is $w_p \rho C_p [T(z_i) + \Delta T_p - T_S]$, and this heat is added in Eq. (7) by a source term of the form

$$S(z) = \frac{2w_p[T(z_i) + \Delta T_p - T_S]}{a(\pi)^{1/2}} \exp\left[-\left(\frac{z - z_x}{a}\right)^2\right] \qquad (14)$$

where a is a length scale for the distribution of the source term. The factor $2/\pi^{1/2}$ arises so that the integral of the source term over depth will yield the total additional heat to be added. In the present paper, when the plume reaches the surface, the explicit heat source term given by Eq. (14) is incorporated into Eq. (7)

$$\partial T/\partial t = (\partial/\partial z)(K_H \partial T/\partial z) - w_p \partial T/\partial z + S(z) \qquad (15)$$

Interpretations similar to those given in Eqs. (9) and (13) can be given for the source term $S(z)$ also.

It may be pointed out that within the approximations involved in the present model, Eqs. (14) and (15) will also apply to a surface outfall.

Numerical Results and Discussions

The equations described in the previous section have been solved numerically for a number of input conditions. The purpose of the calculations was to determine the effects on the over-all thermal structure produced by discharging a heated effluent into the hypolimnion of a body of water. In the present section results for the cases in which thermal discharges are present are compared with the results for the base (that is, without thermal discharges) calculation. The comparisons show many important features which thermal discharge into the hypolimnion can produce. Most of these features have not been discussed in quantitative terms before.

In all the calculations presented in this section, the heat flux q_p due to the power-plant waste heat has been taken to be 280 Btu/ft^2-day. The heat flux added by the power plant has to be compared with the heat flux at the surface of the lake in the absence of the thermal discharge, which for the specific example considered in the previous section (corresponding roughly to conditions in Cayuga Lake, New York), has a maximum value of about 2000 Btu/ft^2-day.[3,5] For Cayuga Lake, 280 Btu/ft^2-day corresponds to the waste heat that would be rejected by a 3500 Mw nuclear power plant.

Heat can be discharged at a specified rate in a variety of ways, depending upon the flow rate of water used for cooling. As discussed before, the heat added per unit surface area of the lake per unit time is $q_p = (Q_p/A)\rho C_p \Delta T_p$. For a fixed heat flux q_p the temperature increase ΔT_p produced by the waste heat is inversely proportional to the volumetric flow rate Q_p of cooling water. Therefore, specification of the heat flux q_p and of the volumetric flow rate or of the pumping velocity $w_p = Q_p/A$ within the lake determines the temperature increase. The temperature increase of the effluent water over that of intake water determines the level at which the discharge spreads horizontally within the lake and therefore the details of the change in the thermal structure produced by the power plant. Two flow-rate and temperature-increase combinations for the specified heat flux have been considered: in one case the pumping velocity was taken to be $\frac{1}{4}$ ft/day with a temperature increase of 10°C at discharge and for the other case $w_p = \frac{1}{6}$ ft/day and $\Delta T_p = 15$°C.

A comparison between temperature profiles without and with thermal discharge at three times during the year is shown in Fig. 4. For the case considered, the thermal discharge has a

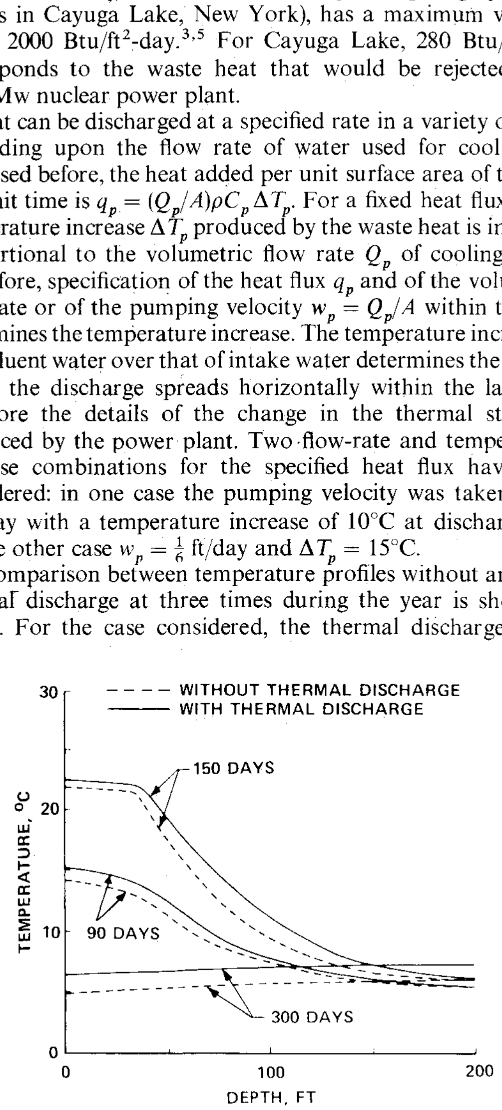

Fig. 4 Effect of thermal discharge on vertical temperature distributions.

pumping velocity of $w_p = \frac{1}{4}$ ft/day and a discharge-temperature increase of $\Delta T_p = 10°C$. The first set of plots, at 90 days, is chosen to show the temperature change after formation of the thermocline during the heating portion of the annual cycle. The curves show that the maximum temperature increase occurs at the surface and that smaller increases occur at greater depths. This behavior is to be expected for the following reason. The intake depth for this calculation has been placed at about 125 ft. At this depth the temperature has varied from its initial value of 2.9°C to around 6°C during the 90 days. With a discharge temperature 10°C above the temperature at the intake depth, the effluent is found to surface throughout the first 90 days. Consequently, the temperature increase produced by the discharge above the natural temperature will be expected to be greatest at the surface. At some time after 90 days the effluent will no longer surface, but will be trapped by buoyancy effects below the surface during part of the stratification period.

The second set of plots in Fig. 4, at 150 days, occurs before the cooling portion of the annual cycle begins. The mixing above the thermocline is very complete both without and with the thermal discharge. In this region, above about 50 ft, the temperature increase produced by the thermal discharge is smaller than in a good portion of the hypolimnion.

The last set of curves, for 300 days, occurs during the period of greatest cooling within the annual cycle. The cooling causes efficient convective mixing, which produces nearly uniform temperature distributions both without and with the thermal discharge. The temperature increase resulting from the thermal discharge is nearly 1.5°C. The actual variations of the temperatures at the surface and at 100 ft depth (for the cases with and without thermal discharges) are shown in Fig. 5.

In Fig. 6, the thermocline depths for the cases with and without thermal discharges are shown as a function of time. The depth of the thermocline is defined for the computer program as the position at which the magnitude of the temperature gradient is maximum. With the effects of the thermal discharge included a thermocline, according to this definition, immediately forms below the surface as a result of the heated effluent. The depth at which this thermocline forms depends upon the details of the heat-source term used to model the discharge; later, however, when a true thermocline forms, the thermocline depth no longer depends upon these details. The position of the thermocline discussed below will refer to the depth determined during this latter period.

With the effects of the thermal discharge included, the thermocline is found to occur at a greater depth than it does naturally for most of the stratification period. This result cannot be simply predicted a priori. The two changes produced by a thermal discharge, namely the heat added and the pumping work performed, have opposite effects on the thermocline.[5] The additional heat tends to increase the temperature, temperature gradients and therefore the stability within the lake. On the other hand, the pumping work tends to enhance mixing the lake. The former effect would, by itself, decrease the depth of the thermocline while the latter would increase its depth. A detailed discussion of these two competing effects and of their relationship to the Monin-Obukhov length has been given in Ref. 5. An examination of the Fig. 6 also reveals that the thermal discharge lengthens the stratification period.

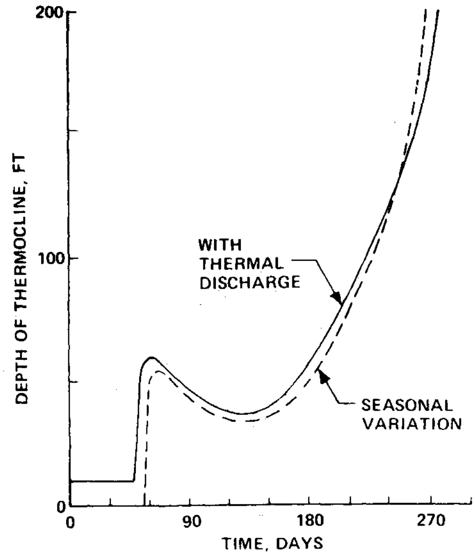

Fig. 6 Effects of thermal discharge on depth of thermocline.

In Fig. 7 the effects of thermal discharges on the turbulent thermal diffusivity are shown. The thermal diffusivity at any point is a local measure of the ability of the water to disperse heat. It is determined by the local stability of the lake and by the competing effect of turbulent mixing. Pumping alone increases the dispersive capability of the lake. A thermal discharge introduces competing effects: the pumping work performed by the discharge increases the mixing, while the additional heat introduced near the surface increases the stability.

It should be emphasized that, as pointed out by Sundaram et al.,[5] the primary difficulty of predicting the effects of heated effluents on the thermal structure of a stratified lake is that the discharges affect the thermal structure of the lake not only directly, but also indirectly by their influence on the turbulence structure in the lake. The effects of the thermal discharges on the structure of the turbulence, and the resulting changes in the eddy diffusivities, are quite important since the manner in which the added heat is dispersed into the lake is controlled by the values of the eddy diffusivities. Thus one cannot assume, as has been done by some authors,[21,22] that the thermal discharges do not affect the mechanisms of epilimnial mixing. This fact becomes clearly evident from an inspection of Fig. 7.

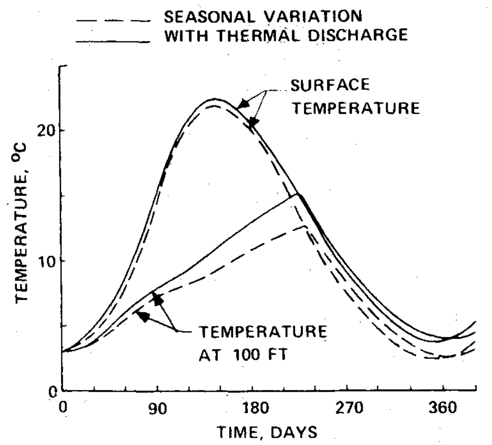

Fig. 5 Effects of thermal discharge on the stratification cycle.

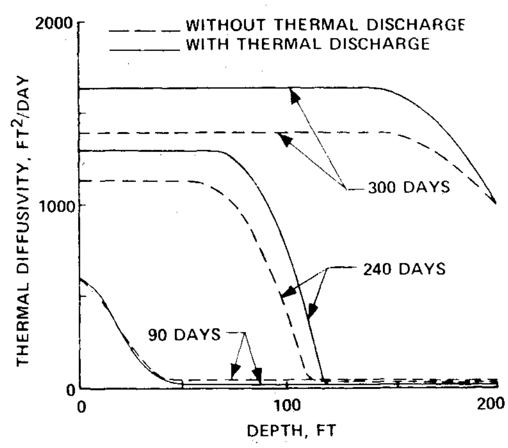

Fig. 7 Effect of thermal discharge on thermal diffusivity.

Table 1 Temperature difference produced by thermal discharges with different discharge temperatures but the same heating rate

	Days	ΔT_S(°C) $w_p=\frac{1}{4}$	ΔT_S(°C) $w_p=\frac{1}{6}$	ΔT_{50}(°C) $w_p=\frac{1}{4}$	ΔT_{50}(°C) $w_p=\frac{1}{6}$	ΔT_{100}(°C) $w_p=\frac{1}{4}$	ΔT_{100}(°C) $w_p=\frac{1}{6}$	ΔT_{150}(°C) $w_p=\frac{1}{4}$	ΔT_{150}(°C) $w_p=\frac{1}{6}$
MAR	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
APR	30	0.41	0.41	0.34	0.34	0.28	0.28	0.24	0.24
MAY	60	0.82	0.84	0.76	0.75	0.46	0.45	0.29	0.28
JUN	90	1.06	1.22	1.22	1.13	0.54	0.46	0.22	0.20
JUL	120	0.58	0.84	1.65	1.46	1.07	0.80	0.31	0.24
AUG	150	0.50	0.67	1.63	1.70	1.75	1.31	0.55	0.41
SEP	180	0.64	0.84	1.05	1.25	2.30	1.77	0.87	0.65
OCT	210	1.10	1.17	1.16	1.22	2.68	2.05	1.23	0.91
NOV	240	1.62	1.51	1.62	1.52	1.67	1.54	1.30	0.93
DEC	270	1.42	1.25	1.46	1.30	1.47	1.30	1.39	1.22
JAN	300	1.52	1.38	1.45	1.31	1.39	1.26	1.36	1.23
FEB	330	1.50	1.39	1.43	1.33	1.37	1.27	1.33	1.23
MAR	365	1.35	1.28	1.38	1.31	1.39	1.31	1.39	1.31

The first set of curves in Fig. 7, at 90 days, shows that with increasing depth the thermal diffusivity including the effects of thermal discharge is first somewhat larger and then smaller than the natural thermal diffusivity. The second set of curves shown are for 240 days. At this time convective turbulence due to cooling, as well as wind-induced turbulence, is producing mixing. The thermal diffusivity with the effects of thermal discharge is considerably larger than the natural diffusivity. At this time a very poor approximation to the diffusivity with thermal-discharge effects included is provided by the natural diffusivity. The last set of curves show the thermal diffusivities at 300 days. At this time the lake is mixed throughout. The thermal diffusivity with discharge effects is largest now.

All the calculations presented above have been for a pumping rate corresponding to $w_p = (\frac{1}{4})$ ft/day and $\Delta T_p = 10$°C. In order to investigate the effects of increased discharge temperatures and decreased pumping rates, calculations have also been carried out for $w_p = (\frac{1}{6})$ ft/day and $\Delta T_p = 15$°C.

The qualitative effects produced by the thermal discharge with $w_p = (\frac{1}{6})$ ft/day and $\Delta T_p = 15$°C are the same as those described above, with one exception. A discharge-temperature increase of 15°C above the temperature at the intake level forces the effluent to remain surfaced throughout the annual cycle.

The quantitative effects are summarized in Table 1. In this Table the increases in temperature produced by the thermal discharge with $w_p = (\frac{1}{4})$ ft/day are listed along with the increases produced when $w_p = (\frac{1}{6})$ ft/day. The important feature common to both calculations is that the additional heat loads introduced by the thermal discharges are the same. The difference between the two results from the change in pumping work done. For the larger pumping velocity $w_p = (\frac{1}{4})$ ft/day, more mixing work is introduced.

During the heating portion of the cycle the increased mixing produced with $w_p = (\frac{1}{4})$ ft/day results in larger temperature increases in the deeper waters and correspondingly smaller temperature increases in the shallower waters. In each case a temperature increase over the naturally occurring temperature can be expected as a result of the heat added by the thermal discharge. However, for $w_p = (\frac{1}{4})$ ft/day, heated waters near the surface are more effectively mixed into the deeper waters by the larger pumping velocity.

During the cooling portion of the cycle, when the lake is thoroughly mixed, the temperature increase throughout is larger for the case when $w_p = (\frac{1}{4})$ ft/day. Once again the increased mixing resulting from the larger pumping rate explains this feature. The rate of heat added to the lake at any time is equal to the heat flux $K(T_E - T_S)$ at the lake surface times the surface area A of the lake plus the rate of heat added by the discharge. Under periodic conditions, i.e., when the lake has adjusted to the increased thermal load provided by the discharge, the integral over an annual cycle of the heat added at the surface plus the heat added by the discharge must be zero. Thus

$$K \oint (T_E - T_S)\, dt = -Q$$

where Q is the total heat added over a year by the thermal discharge and where K is the constant heat transfer coefficient at the lake surface. For both cases discussed above, the thermal discharge with $w_p = (\frac{1}{4})$ ft/day and the discharge with $w_p = (\frac{1}{6})$ ft/day, the total heat added is $280AT$, where A is the surface area of the lake and T is one year. Also, for both pumping rates the equilibrium temperature variation is the same. However, as discussed above, the larger pumping rate produces a lower surface temperature during the heating portion of the annual cycle. Therefore, since the integral of

$$\oint (T_E - T_S)\, dt$$

is the same for both cases and since T_S is lower during stratification for the higher pumping rate, this temperature must be higher during the cooling portion of the cycle. During cooling, however, the lake is thoroughly mixed so that the surface temperature is very nearly the temperature throughout.

Concluding Remarks

When the waste heat from electric generating plants is discharged into a stratified lake, the effluents affect the seasonal stratification cycle of the lake in a variety of ways. Thus the discharges influence the thermal structure of the lake directly, as well as through their effects on the structure of the wind-induced turbulence. Moreover, when the intake is located in the hypolimnion of the lake, the transfer of large quantities of water from one level to another (which is a necessary consequence of the thermal discharges) also results in a change in the potential energy of stratification of the lake. Indeed, under certain conditions, this effect of "pumping" may be a dominant one. In the present paper, a theoretical model which includes all of the above effects was described. This model provides the means, hitherto unavailable in the literature, of assessing the relative thermal effects of various modes of discharges.

The theoretical model described in the present paper is essentially an extension of the theoretical model formulated earlier by the authors to study the mechanism of formation and maintenance of thermoclines, as well as the stratification cycle of temperature lakes. The important characteristic feature of the model is that the influence on the eddy diffusivities of the interactions between wind-induced turbulence and the stable buoyancy gradients due to surface heating (as well as the thermal discharges) is included explicitly. Although the nonlinear aspects of these interactions make the problems difficult, the nonlinearity is an essential feature, and as such, it has been retained in the model. It should be pointed out that if the nonlinear aspects of the effects of the thermal discharges are neglected (that is, if it is assumed that the mechanisms of turbulent mixing within the lake are unaffected by the thermal discharges), then the problem of predicting the effects of thermal discharges becomes a relatively simple one.

In the present paper, the theoretical model has been used to study the effects of thermal discharges at or below the level of the thermocline on the seasonal stratification cycle of temperature lakes. The major advantage of a hypolimnetic discharge is that the effluents can be trapped below the thermocline at least during part of the stratification season, so that surface effects of the discharges can be expected to be minimized. The calculations presented in the paper show that the increases in temperatures, due to the thermal discharges, are indeed greater at deeper levels than at the surface. However, some increase in the average surface temperature of the lake is unavoidable, since such an increase is essential (once the lake attains a new thermal equilibrium) for the lake to dissipate the added power-plant heat to the atmosphere.[5]

The calculations reported in the present paper also lead to several general conclusions. A thermal discharge increases the temperatures at all depths and at all times over those occurring naturally. However, the relative magnitudes of the increases at various depths and times are dependent upon the specific mode of discharge. The thermal discharges also increase the length of the stratification period. For the specific calculations performed during the present study, the depths of the thermocline were found to be somewhat larger for the cases with thermal discharges than for those without. However, under certain conditions, thermal discharges may lead to a reduction in the epilimnetic volume.

The results described in the present paper are by no means a complete assessment of the effects of thermal discharges on stratified lakes. The present paper has been concerned only with the effects of some specific modes of discharges. However, the formulations used have included all crucial features of the interactions between the discharges and the thermal structure of the lake, and the techniques employed here can be easily generalized to various other modes of discharges. It is hoped that in the future a parametric study of the effects of various modes of discharges can be carried out.

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